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MULTI-SERVER BATCH SERVICE QUEUEING MODEL
WITH VARIABLE SERVICE RATES

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ABSTRACT

In this paper, we study the batch queueing model, where multi-servers are used to provide the service. Arrival follows a Poisson distribution and service time is exponentially distributed. If there are less than the batch of B customers, they are served individually. However, if there are the batch of B customers, all the customers are served together. We calculate the probability distribution and the performance measures of queue length and waiting time for both conditions: having less than the batch of B customers and having more than the batch of B customers using recursive method. Finally, we verify the results with some numerical illustrations presenting some applications in the real life situation.

KEYWORDS: Batch, Customer, Performance, Queueing, Server

1. INTRODUCTION

Mostly, customers go to get the service in their own time and convenience, individually. But, this rule may not work in all the situations. In some conditions, the server provides the service for a group of customers instead of serving individually. In this paper, we assume a model where a batch of B customers are served at a time. There are certain number of servers providing the service to the customers. If customers are enough to make a group of batch B, they all are taken together for the service considering them as a single customer. Instead, if customers are less than the batch B, they are served individually with the different service rate η. This type of queueing model is applicable in the field of transportation, which moves only for the fixed number of passengers. An elevator serves a group of 4 or 5 persons at a time, but if there are less people then also it serves. A cable car also prefers to serve up to its capacity, but that is forced to carry even if there are less passengers. These are some examples of this model applied in the real life. Several researches have performed in the field of the batch queuing system and some of them are observed here onwards in this section.

Saha andAlfa [1] studied discrete-time two-phase packet arrival queuing system with batch service in the first queue and individual service in the second queue. They calculated the queue length, average waiting time of packets and the effect on batch size on the waiting time. Banergee et al. [2] dealt with a finite-buffer queue having a single server batch of variable size to obtain the average number of packets waiting in the queue and in the system. They calculated the server rejection probabilities also and verified their findings by means of numerical results. Gupta and Sikder [3] considered a single server finite-buffer bulk-service queue following exponential inter-arrival and arbitrary service time distribution. Minimum batch size a and maximum batch size b is needed for the server to serve. They obtained distributions of the number of customers in the queue at arbitrary service completion and vacation termination epochs along with some key

performance measures. Ghimire et al. [4, 5] calculated some performance measures for a bulk queueing model of a fixed batch size b and finite capacity queueing model considering time dependent arrival and service rate, respectively. In both cases, they verified the results graphically using MATLAB programming. Balbo and Vigliotti [6] analyzed a batch service queue, in which, whole batch or single customer had been served based on the number of customers available in the system. They introduced quasi-reversible queues for the computationally efficient approximations. All the results were compared for different parameter settings using both stochastic order arguments and numerical experiments. Baruah et al. [7,8] studied batch arrival queueing system of variable batch size following essential and optional service provision. Later, they derived a two stage queuing model with reneging where server is unavailable during the system breakdown or vacation periods. The supplementary variable technique was used to calculate the mean queue length and mean waiting time. Yu et al. [9] investigated batch arrival queue under N-policy, with single vacation and setup times, in which, server is turned off one time for vacation of random length V, if the system became empty and resumes after setup time U, if the queue length met the threshold N ($N \geq 1$). They calculated, the system size distribution and the queue length distribution together with some numerical examples for the verification.

Sikder and Gupta [10] contributed in the field of batch arrival and batch service queue with single and multiple vacations. They calculated performance measures like average queue length, average waiting time, probability of busy server, blocking probabilities etc. Parveen and Begum [11] studied a single server bulk service queue with general arrival pattern and multiple working vacation period, using Embedded Markov Chain technique. They derived steady state probability distribution at pre arrival epoch and arbitrary epoch presenting some numerical examples graphically. Shinde and Patankar [12] considered balking, reneging and multiple vacations in a state dependent bulk service queue. The server was supposed to wait for some time, if there were the waiting customers less than the batch size a in the queue, called change over time. If no customer arrived during changeover time, servers go for vacation. Performance measures like average queue length, expected waiting time in queue and in the system, variance and cost analysis were calculated. Bountali and Economou [13] considered batch service queueing system to study the joining or balking dilemma based on the information provided upon arrival called unobservable and observable. They presented numerica lexperiments to investigate the differences in the behaviour of customers in batch service systems and single service systems. Ziani et al. [14] dealt with batch arrivals of two customers to study their strategic behaviour in a single server Markovian $M/M/1$ queue. In their model, customers join the queue or balk depending on his/her own decision, on his/her partner's decision and of the system state. They compared the cases, when information about the system state was provided, and when it was not provided. Sah and Ghimire [15] studied transient Erlangian queueing system to calculate the mean queue length and expected waiting time distribution using probability generating function and Laplace transform method. Wu [16] performed his research about the importance of batch in manufacturing systems considering three types of batches namely transfer batches, parallel batches and serial batches. He classified different batching behaviour and verified the validity by simulation.Yu and Alfa [17] presented a discrete-time single-server finite-buffer queue with Markovian arrival process and generally distributed batch-size-dependent service time. They obtained appropriate phase-type distribution, the joint state probabilities at various time epochs using matrix analytic method and embedded Markov chain technique.

In all these literatures, it can be observed that different algorithms have been described with many propositions. Some of the researchers have performed their research of batch in the arrival process, whereas some studied batch in service facility. Some are found interested in both batch arrival and batch service, but almost all of them have used a single server model. Balking and reneging in the batch arrival and service are the next areas, some of the authors are interested in.

Here, we put our efforts in the batch service with multiple servers in which a group which cannot make a batch of B customers also get the service in the different service rate. If there are less than the batch of B customers, they are served individually with service rate η. However, the batch of B customers is served with the rate of μ. We assume p as the probability that a batch of size B is served and they leave the system with rate $\delta = \mu * p$. Then $q = 1 - p$ is the probability that a single customer is removed from the queue after the service completion. So, $v = \mu * q$ is the rate at which single customer is removed from the system when more than the batch of B customers are present.

The rest of the paper is organized as follows: Section 2 presents some of the assumptions and notations used in the model along with mathematical derivation. Section 3 includes the performance measures relating to some established results of different models, whereas Section 4 consists of the numerical results and verification of the model using simulation. Section 5 concludes the paper, including some applications of the model.

2. MATHEMATICAL MODEL

In this Section, we establish the mathematical model with some notations and assumptions using the transition diagram for the balanced equations. The followings are some of the assumptions and notations we have considered in this model.

(i) λ = Arrival rate of a new customer.

(ii) μ =Total service rate of the system when loaded by the batch of B customers

(iii) η_i= Service rate for the customers when loaded by less than the batch of B customers.

(iv) p = Probability of the service completion for the batch of B customers.

(v) q =Probability of the service completion of individual customers when at least B customers are in the queue; $p + q = 1$

(vi) $v = \mu * q$ = Rate at which single customer leaves the system when more than B customers are present.

(vii) $\delta = \mu * p$ = Rate at which a batch of B customers leaves the system.

(viii) P_n = Stationary probability of finding n customers in the system$(n \geq 0)$.

To represent these assumptions, probability transition diagram for different states have been shown in Figure 1 (see [6] also). We assume the constant arrival rate throughout the system, but service rate differs based on the number of customers.
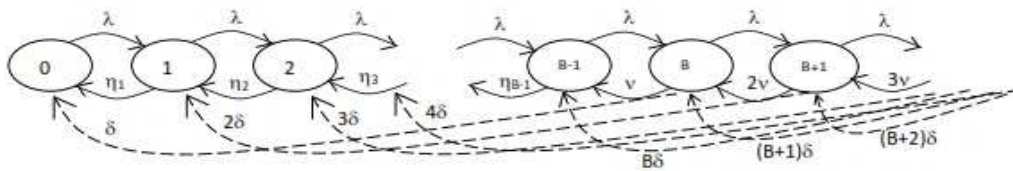

Figure 1: Transient Diagram

Steady state equations can be derived fromFigure 1. These equations are solved using recursive method. All the probabilities of other states are converted in the form of P_0, which ultimately helps to calculate the total probability P_n.

The system is separated into two cases: (i) when there are the customers less than the batch size B and (ii) when customers are greater or equal to the batch size B. The equations are derived as follows:

$$\lambda P_0 = \eta_1 P_1 + \delta P_B \tag{1}$$

$$(\lambda + \eta_1)P_1 = \eta_2 P_2 + \lambda P_0 + 2\delta P_{B+1} \tag{2}$$

$$(\lambda + \eta_2)P_2 = \eta_3 P_3 + \lambda P_1 + 3\delta P_{B+2} \tag{3}$$

continuing this way

$$(\lambda + \eta_{B-2})P_{B-2} = \eta_{B-1}P_{B-1} + \lambda P_{B-3} + (B-1)\delta P_{2B-2} \tag{4}$$

$$(\lambda + \eta_{B-1})P_{B-1} = \nu P_B + \lambda P_{B-2} + B\delta P_{2B-1} \tag{5}$$

$$(\lambda + \nu + \delta)P_B = 2\nu P_{B+1} + \lambda P_{B-1} + (B+1)\delta P_{2B} \tag{6}$$

$$(\lambda + 2\nu + 2\delta)P_{B+1} = 3\nu P_{B+2} + \lambda P_B + (B+2)\delta P_{2B+1} \tag{7}$$

$$[\lambda + (k+1)\nu + (k+1)\delta]P_{B+k} = (k+2)\nu P_{B+k+1} + \lambda P_{B+k-1} + (B+k+1)\delta P_{2B+k} \tag{8}$$

Rewriting the above equations, we have

$$\eta_1 P_1 = \lambda P_0 - \delta P_B \tag{9}$$

$$\eta_2 P_2 = \lambda P_1 - \delta(P_B + 2P_{B+1}) \tag{10}$$

$$\eta_3 P_3 = \lambda P_2 - \delta(P_B + 2P_{B+1} + 3P_{B+2}) \tag{11}$$

$$\eta_{B-1}P_{B-1} = \lambda P_{B-2} - \delta \sum_{j=1}^{B-1} j P_{(B-1)+j} \tag{12}$$

$$(\nu + \delta)P_B = \lambda P_{B-1} - \delta \sum_{j=1}^{B-1}(j+1)P_{B+j} \tag{13}$$

$$(2\nu + 2\delta)P_{B+1} = \lambda P_B - \delta \sum_{j=1}^{B}(j+1)P_{B+j} \tag{14}$$

$$(3\nu + 3\delta)P_{B+2} = \lambda P_{B+1} - \delta \sum_{j=1}^{B+1}(j+1)P_{B+j} \tag{15}$$

$$((k+1)\nu + (k+1)\delta)P_{B+k} = \lambda P_{B+k-1} - \delta \sum_{j=1}^{B+k-1}(j+1)P_{B+j}, \quad k = 0,1,2,3,\ldots \tag{16}$$

Without loss of generality, let us define

$$\frac{\sum_{j=1}^{i+1} j P_{B-1+j}}{P_i} \delta = \phi_i, \quad 0 \le i \le B-2 \tag{17}$$

And $\frac{\sum_{j=1}^{B-1}(j+1)P_{n+1+j}}{P_n} \delta = \psi_n, \quad n \ge B-1 \tag{18}$

Then, the above set of equations reduce to

$$P_1 = P_0 \frac{(\lambda - \phi_0)}{\eta_1} \tag{19}$$

$$P_2 = P_1 \frac{(\lambda - \phi_1)}{\eta_2} \tag{20}$$

$$P_3 = P_2 \frac{(\lambda - \phi_2)}{\eta_3} \tag{21}$$

$$P_{B-1} = P_{B-2} \frac{(\lambda - \phi_{B-2})}{\eta_{B-1}} \tag{22}$$

$$P_n = P_{n-1} \frac{(\lambda - \psi_{n-1})}{jv + j\delta}, \quad n \geq B \tag{23}$$

These equations lead to the solution:

$$P_n = \begin{cases} P_0 \left[\prod_{i=1}^n \frac{(\lambda - \phi_{i-1})}{\eta_i} \right], & n \leq B - 1 \\ P_0 \left[\prod_{i=1}^{B-1} \frac{(\lambda - \phi_{i-1})}{\eta_i} \right] \left[\frac{\prod_{j=B}^n (\lambda - \psi_{j-1})}{\prod_{j=1}^{B-1}(jv + j\delta)} \right], & n \geq B \end{cases} \tag{24}$$

Therefore, the system of equations can be written as

$$P_n = \begin{matrix} P_{n-1} \frac{(\lambda - \phi_{n-1})}{\eta_n}, & 1 \leq n \leq B - 1 \\ \dots \dots \\ P_{n-1} \frac{(\lambda - \psi_{n-1})}{jv + j\delta}, & n \geq B \end{matrix} \tag{25}$$

Then, we get the geometric distribution

$$P_n = \begin{cases} P_0 [\prod_{i=1}^n \sigma_i], & n \leq B - 1 \\ P_0 [\prod_{i=1}^{B-1} \sigma_i] \rho^{n-B+1}, & n \geq B \end{cases} \tag{26}$$

Where, $\sigma_i = \frac{(\lambda - \phi_{i-1})}{\eta_i}, \; 1 \leq i \leq B - 1$

$$\rho = \frac{\lambda - \psi_n}{jv + j\delta}, \quad j \geq 1$$

For the explicit expression of the model, let us consider

$$\psi_n = \frac{\sum_{j=1}^{B-1}(j+1)P_{n+1+j}}{P_n} \delta, \quad n \geq B - 1 \tag{27}$$

$$= \frac{\sum_{j=1}^{B-1}(j+1) P_0 [\prod_{i=1}^{B-1} \sigma_i] \rho^{n+1+j-B+1}}{P_0 [\prod_{i=1}^{B-1} \sigma_i] \rho^{n-B+1}} \delta$$

$$= \frac{[\prod_{i=1}^{B-1} \sigma_i] \sum_{j=1}^{B-1}(j+1) \rho^{n-B+1} \rho^{j+1}}{[\prod_{i=1}^{B-1} \sigma_i] \rho^{n-B+1}} \delta$$

$$\text{Therefore,} \psi_n = \sum_{j=1}^{B-1} (j+1) \rho^{j+1} \delta$$

This expression is free from n, so we can write $\psi_n = \psi$ and $\psi = \sum_{j=1}^{B-1}(j+1) \rho^{j+1} \delta$ is a constant for all n

Hence, $\rho = \frac{\lambda - \psi}{jv + j\delta}$

From these equations, we can get that

$$P_0 = \left[1 + \sum_{n=1}^{B-1} [\prod_{i=1}^n \sigma_i] + [\prod_{i=1}^{B-1} \sigma_i] \frac{\rho}{1-\rho} \right]^{-1} \tag{28}$$

To get the value of P_0, we have to know the value of σ_i and we follow the following procedure to calculate it.

From equation (26), we can write

$$\sum_{j=1}^{B-1}(j+1)P_{n+j} = P_0[\prod_{i=1}^{B-1}\sigma_i]\sum_{j=1}^{B-1}(j+1)\rho^{n-B+1+j}$$

$$= P_0[\prod_{i=1}^{B-1}\sigma_i]\rho^{n-B+1}\sum_{j=1}^{B-1}(j+1)\rho^{j}$$

$$= P_n\frac{B\rho^{B+1}-(B+1)\rho^B-\rho^2+2\rho}{(1-\rho)^2} \tag{29}$$

We have, $P_n = \rho P_{n-1}$, then using these two conditions, we can write the equation (12) as

$$\eta_{B-1}P_{B-1} = \lambda\frac{P_{B-1}}{\sigma_{B-1}} - \delta P_{B-1}\sum_{j=1}^{B-1}j\rho^{j}$$

$$\Rightarrow \eta_{B-1} = \frac{\lambda}{\sigma_{B-1}} - \delta\frac{(B-1)\rho^{B+1}-B\rho^B+\rho}{(1-\rho)^2}$$

Therefore, $\sigma_{B-1} = \dfrac{\lambda(1-\rho)^2}{\eta_{B-1}(1-\rho)^2 + \delta\{(B-1)\rho^{B+1}-B\rho^B+\rho\}}.$

Using the similar technique, we can get the value of σ_i for $i = B-2, B-1, \dots, 2, 1$

$$\sigma_i = \frac{\lambda(1-\rho)^2}{\eta_i(1-\rho)^2 + \delta(\prod_{k=i+1}^{B-1}\sigma_k)\{i\rho^{i+1}-(i+1)\rho^i-\rho^2+2\rho\}}.$$

3. PERFORMANCE MEASURE

Here, we observe the two cases, one for the customers having less than the batch size B and the other for more than the batch size B. When there are the customers less than the batch size B i. e. for $n \le B-1$, each of the arrivals are served individually and hence the model is same as the case of $M/M/1$ queueing model following the same performance measures. On the other hand, for $n \ge B$, customers are served in a group or in a batch considering the whole batch as a single arrival. So, this case is similar to the case of $M/M/c$ queueing model. Hence, performance measures for the case $n \ge B$ are same as the performance measures of $M/M/c$ queueing model.

Case I: $n \le B - 1 \left(\rho = \frac{\lambda}{\eta}\right)$

(i) Average number of customers in the queue $L_q = \frac{\rho^2}{(1-\rho)}$;

(ii) Average number of customers in system $L_s = \frac{\rho}{(1-\rho)}$;

(iii) Average waiting time in the system $W_s = \frac{\rho}{\lambda(1-\rho)}$;

(iv) Average waiting time in the queue $W_q = \frac{\rho}{\mu(1-\rho)}$.

Case II: $n \ge B \left(\rho = \frac{\lambda-\psi}{j\nu+j\delta}\right)$

(i) Average number of customers in the queue $L_q = P_0\frac{\left(\frac{\lambda-\psi}{\nu+\delta}\right)^j}{j!}\frac{\rho}{(1-\rho)^2}$;

(ii) Average number of customers in system $L_s = L_q + \rho$;

(iii) Average waiting time in the system $\qquad W_s = \frac{L_s}{\lambda}$;

(iv) Average waiting time in the queue $\qquad W_q = \frac{L_q}{\lambda}$.

It is worthwhile to note that for $j = 1$ in the value of ρ the model is identical to the model in [6]. Similarly, for $B = 1$ and $j = 1$ the model is identical to $M/M/1$ queueing model showing that our model is more general than the models studied earlier by several authors.

4. NUMERICAL RESULTS

We have calculated the performance measures for two different conditions in which, either the customers are more than the batch size or less than that. When customers are less than the batch size B, the case is simple $M/M/1$ queueing model. We have used the MATLAB software to check the validity of the algorithms used in the model only in the second case, having the customers more than the batch size B. There are a total of eight graphs presented to verify the results.

Figure 2: Average Number of Customers in the Queue Vs Arrival Rate

Figure 2 is the graph of average number of customers in the queue against arrival rate which indicates that number of customers in the queue increases for the bigger arrival rate. On the other hand, if the service rate is increased queue length decreases even for the bigger arrival rate.
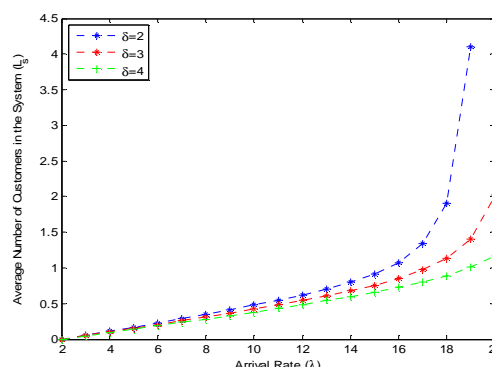

Figure 3: Average Number of Customer in the System Vs Arrival Rate

In Figure 3, it is observed that average number of customers in the system (number of customers in the queue and the one in front of the server) increases for the larger arrival rate showing less number of customers in the system for the faster service rate.
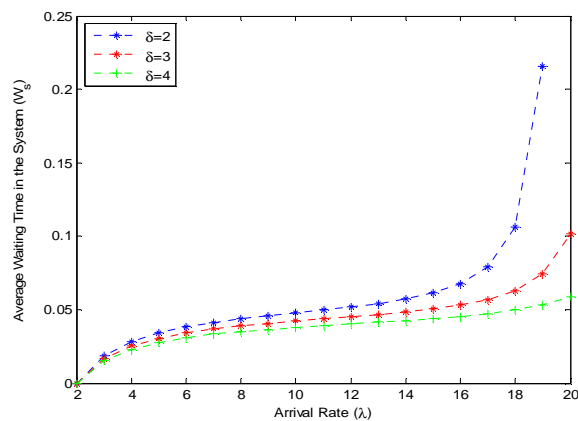
Figure 4: Average Waiting Time in the System Vs Arrival Rate

Figure 4 is the graph for average waiting time Vs arrival rate which shows that customers has to wait longer for the bigger arrival rate. The waiting time can be reduced if the service rate is increased. In all Figure 2, Figure 3 and Figure 4 we can observe that if the service rate is increased, queue length decreases which is realistic in nature.
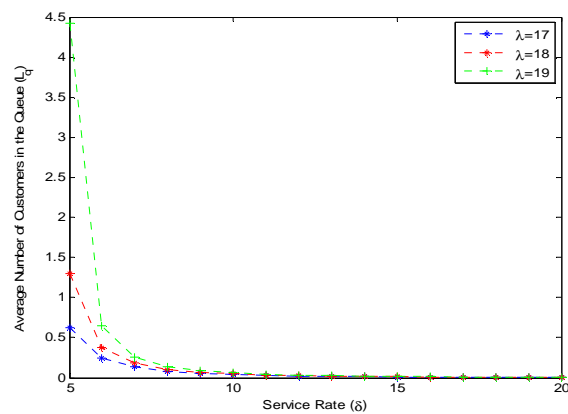

Figure 5: Average Number of Customers in the Queue Vs Service Rate

Figure 5 refers to the graph of average number of customers in the queue Vs service rate for the different arrival rates. From the graph, we notice the increment in the customers' number in the queue for the slower service rate and less number of customers in the queue for the smaller arrival rate.
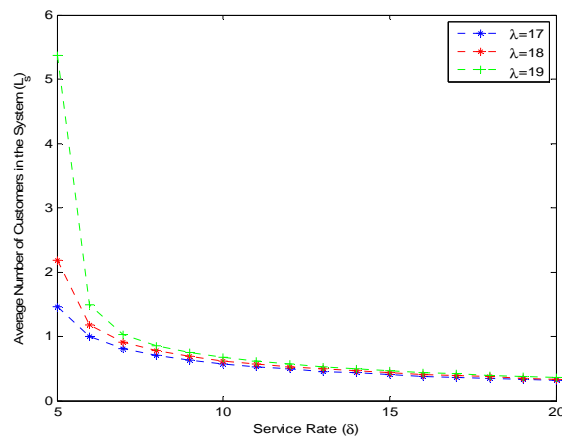

Figure 6: Average Number of Customers in the System Vs Service Rate

Likewise, we have plotted the graph for average number of customers in the system Vs service rate in Figure 6 which indicates that number of customers in the system increases whenever arrival rate increases. However, it is noticed that for the faster service rate number of customers in the system are less in comparison to the slower service rate.
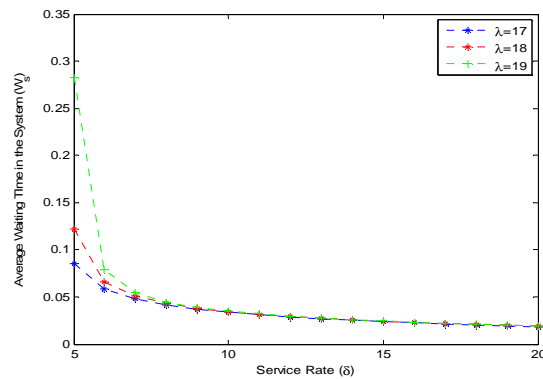

Figure 7: Average Waiting Time in the System Vs Service Rate

Similarly, Figure 7 is plotted average waiting time in the system against the service rate in which three different values of arrival rates are taken to observe the difference in waiting time by the customers in the system. It is noticed that for a faster service rate customer has to wait for shorter period of time whereas more arrival rate results the longer waiting time.
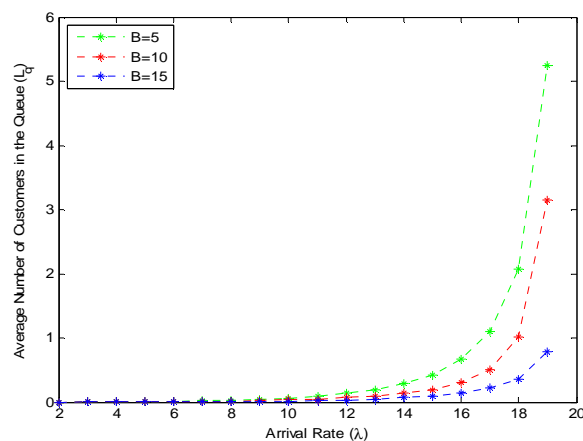

Figure 8: Average Number of Customers in the Queue Vs Arrival Rate

Figure 8 is the graphs for average number of customers in the queue against arrival rate in which smaller batch size refers to the longer queue length and bigger batch size results shorter queue length depending on the arrival rate.
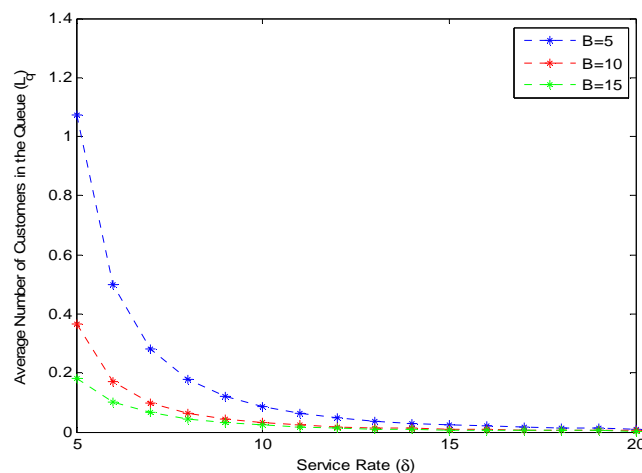
Figure 9: Average Number of Customers in the Queue Vs Service Rate

Finally, average number of customers against service rate is plotted in Figure 9 indicating bigger batch size has shorter queue length according as the service rate increases. All these graphs help us to prove that the proposed model is valid. Some of the areas applicable to this model are explained in the conclusion below.

5. CONCLUSIONS

There is a benefit for the customers as well as the servers in the batch queuing system. Every member of the batch should not necessarily stand in front of the server for the service, because any one member of the group can check the progress of their service. On the other hand, the server can also deal with one of the members of the batch whether or not all the requirements for the service are fulfilled. But, whenever customers come in a group, it is not always necessary that it forms the required batch of the necessary size. On some occasions, it is not even possible to make a group and customer should go alone to get a service. If provision is to provide the service only to a given batch, arrivals should wait until the required batch size is formed. We considered the case to serve the whole group with a fixed service rate, if there is the batch of B customers. If the customers could not reach up to the batch of B, they are served individually with different service rate. There could be some cases where vehicles can run only when it is full because it could meet the operating cost only for the full passengers. In this case,the vehicle could provide the service even for less passengers if the rest of the passengers compensate the fare. This model can be applied in the production companies also. If a company orders a batch of products, the producer may or may not have the ordered quantity. If there are enough number of ordered quantities, they serve and supply. If the producer does not have enough quantity, either the request should be managed with less quantity or the products should be sold to others separately.

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REFERENCES

1. Saha, S & Alfa, A. S. (2008). Selecting batch size in discrete-time two-phase queuing system. *Mathematical and Computer Modelling*, 47, 1246-1253.

2. Banerjee, A., Gupta, U. C. & Goswami, V. (2014). Analysis of finite-buffer discrete-time batch-service queue with batch-size-dependent service. *Computers & Industrial Engineering*, 75, 121-128.

3. Gupta, U.C. & Sikdar, K. (2004). The finite-buffer $M/G/1$ queue with general bulk-service rule and single vacation. *Performance Evaluation*, 57, 199-219.

4. Ghimire, S, Ghimire, R. P. & Thapa, G. B. (2014). Mathematical models of M^b/M/1 bulk arrival queueing system. *Journal of the Institute of Engineering*, 10 (1), 184-191.

5. Ghimire, S, Ghimire, R. P. & Thapa, G. B. (2015). Performance evaluation of unreliable $M(t)/M(t)/n/n$ queueing system. *British Journal of Applied Science & Technology*, 7 (4), 412-422.

6. Balbo, G. & Vigliotti, M. G. (2015). On the analysis of a $M/M/1$ queue with bulk services. *The Computer Journal*, 58 (1), 57-74.

7. Baruah, M., Madan, K. C. & Eldabi, T. (2013). A batch arrival queues with second optional service and engine during active periods. *Revista Investigation Operational*, 34 (3), 244-258.

8. Baruah, M., Madan, K. C. & Eldabi, T. (2013). A two stage batch arrival queues with reneging during vacation and breakdown periods. *American Journal of Operations Research*, 3, 570-580.

9. Yu, Z., Liu, M. & Ma, Y. (2010). Steady-state queue length analysis of a batch arrival queue under N-policy with single vacation and setup times. *Intelligent Information Management*, 2, 365-374.

10. Sikdar, K. & Gupta, U.C. (2008). On the batch arrival batch service queue with finite buffer under server's vacation: $M^X/M^Y/1/N$ queue. *Computers and Mathematics with Applications*, 56, 2861-2873.

11. Parveen, M. J.& Begum, M. I. A. (2013). General bulk service queueing system with multiple working vacation. *International Journal of Mathematics Trends and Technology,* 4 (9), 163-173.

12. Shinde, V. & Patankar, D. (2012). Performance analysis of state dependent bulk service queues with balking, reneging and server vacation. *IJORN*, 1, 61-69.

13. Bountali, O. & Economou, A. (2017). Equilibrium joining strategies in batch service queueing systems. *European Journal of Operational Research*, 260, 1142-1151.

14. Ziani, S., Rahmoune, F. & Radjef, M. S. (2015) Customers' strategic behaviour in batch arrivals $M^2/M/1$ queue. *European Journal of Operational Research*, 247, 895-903.

15. Sah, S. S. & Ghimire, R. P. (2015). Transient analysis of queueing model. *Journal of the Institute of Engineering*, 11 (1), 165-171.

16. Wu, K. (2014). Taxonomy of batch queueing models in manufacturing systems. *European Journal of Operational Research*, 237, 129-135.

17. Yu, M. & Alfa, A. S. (2015). Algorithms for computing the queue length distribution at various time epochs in $DMAP/G^{(1,a,b)}/1/N$queue with batch-size-dependent service time. *European Journal of Operational Research*, 244, 227-239.